Keplerov zadatak

Keplerovi zakoni kretanja planeta

1. Sve planete se kreću oko Sunca po eliptičnim putanjama, a u zajedničkom fokusu (žiži) eliptičnih putanja nalazi se Sunce.
2. Duž koja spaja planete sa Suncem u istim vremenskim intervalima opisuje iste površine.
3. Kvadrat vremena obilaska planete oko Sunca proporcionalan je trećem stepenu njenog srednjeg rastojanja od Sunca.

Jednačina kretanja planeta

$$\mu = \frac{Mm}{m+M} = m$$

$$m = 5{,}99 \cdot 10^{24}\, kg$$

$$M = 1{,}99 \cdot 10^{30}\, kg$$

$$\vec{F} = -\frac{\gamma Mm}{r^2}\vec{r}_0 = -\frac{\gamma Mm}{r^3}\vec{r}$$

$$\gamma = 6{,}67 \cdot 10^{-11}\, \frac{m^3}{kg \cdot s^2}$$

$$L_z = (\vec{r} \times m\vec{v})_z = m(xv_y - yv_x)$$

$$E = \frac{1}{2}mv^2 - \frac{\gamma Mm}{r}$$

$$m\frac{d^2\vec{r}}{dt^2} = -\frac{\gamma Mm}{r^3}\vec{r}$$

$$F_x = -\frac{\gamma Mm}{r^2}\cos\Theta = -\frac{\gamma Mm}{r^3}x$$

$$F_y = -\frac{\gamma Mm}{r^2}\sin\Theta = -\frac{\gamma Mm}{r^3}y$$

$$\frac{d^2x}{dt^2} = -\frac{\gamma M}{r^3}x$$

$$\frac{d^2y}{dt^2} = -\frac{\gamma M}{r^3}y$$

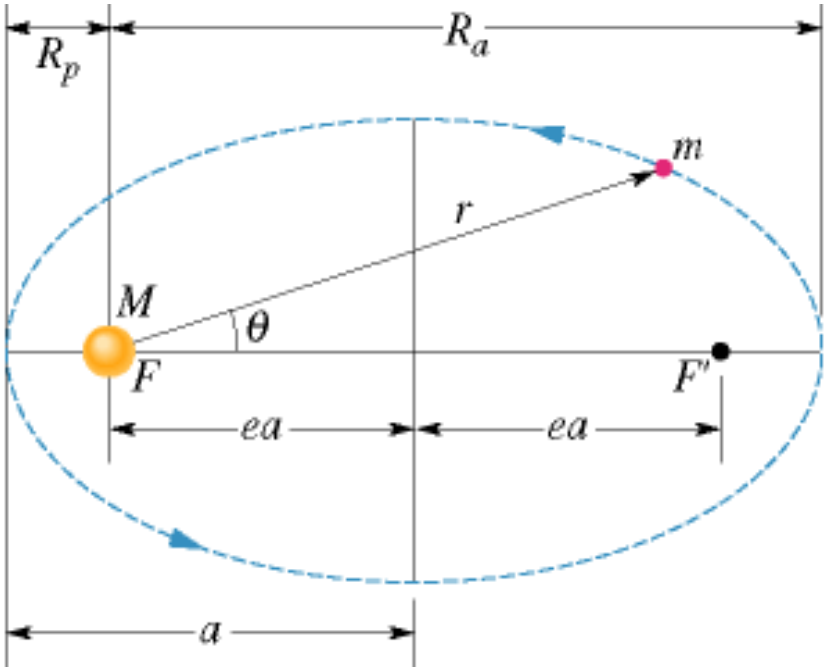
Kružna orbita

$$a = \frac{v^2}{r}$$

$$m\frac{v^2}{r} = \frac{\gamma M m}{r^2} \Rightarrow v = \sqrt{\frac{\gamma M}{r}}$$

$$T = \frac{2\pi r}{v} \Rightarrow T^2 = \frac{4\pi^2 r^3}{\gamma M}$$

Eliptična orbita



$$Fm + F'm = 2a$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$0 < e < 1$$

$$b = a \Rightarrow e = 0$$

Astronomska jedinica

$$1AU = 1.496 \cdot 10^{11} m$$

$$1godina = 3.15 \cdot 10^{7} s$$

$$T = 1godina$$

$$a = 1AU$$

$$\gamma M = \frac{4\pi^2 a^3}{T^2} = 4\pi^2 \frac{AU^3}{godina^2}$$

$$1.5AU = 2{,}244 \cdot 10^{11} m$$

Numeričko modeliranje putanje

$$accel(i) = -GM * pos(i)/(r*r*r)$$

$$vel(i) = vel(i) + accel(i) * dt$$

$$pos(i) = pos(i) + vel(i) * dt$$

Program planet

```
# include <stdio.h>
# include <math.h>

main ()
{
        FILE *fp1;
        fp1=fopen("planet.data","w");
        double x,y,vx,vy,r,ax,ay,gm,t,dt;

        t=0;
        x=1;
        y=0;
        vx=0;
        vy=6.28;
        gm=4*pow(3.14159,2);
        dt=0.001;
```

```
while (t <= 2)
{
                r=sqrt(x*x+y*y);

    ax=-gm*x/pow(r,3);
    ay=-gm*y/pow(r,3);
    vx=vx+ax*dt;
     vy=vy+ay*dt;
     x=x+vx*dt;
    y=y+vy*dt;
    t=t+dt;
            fprintf(fp1,"%10.3f,%10.3f,\n",x,y);
}
fclose(fp1);
}
```

Zemaljska jedinica (EU)

$$1EU = 6.37 \cdot 10^{6} m$$

$$G = 6.67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \left(\frac{1EU}{6.37 \cdot 10^{6} m}\right)(3.6 \cdot 10^{3} s/h) =$$

$$= 3.34 \cdot 10^{-24} \frac{EU^3}{kg \cdot h^2}$$

$$Gm = 3.34 \cdot 10^{-24} \frac{EU^3}{kg \cdot h^2} \cdot 5.99 \cdot 10^{24} kg = 20. \frac{EU^3}{h^2}$$